12.3 THE OIL-DROP EXPERIMENT

In 1906 Robert A. Millikan, then a lowly assistant professor at the University of Chicago, devised an ingenious experiment that for the first time made it possible to measure the charge on an individual droplet rather than on a cloud. Through Millikan's experiment it became possible to determine whether or not electricity in gases and chemical solutions is built out of electrons and whether each electron has the same amount of charge.

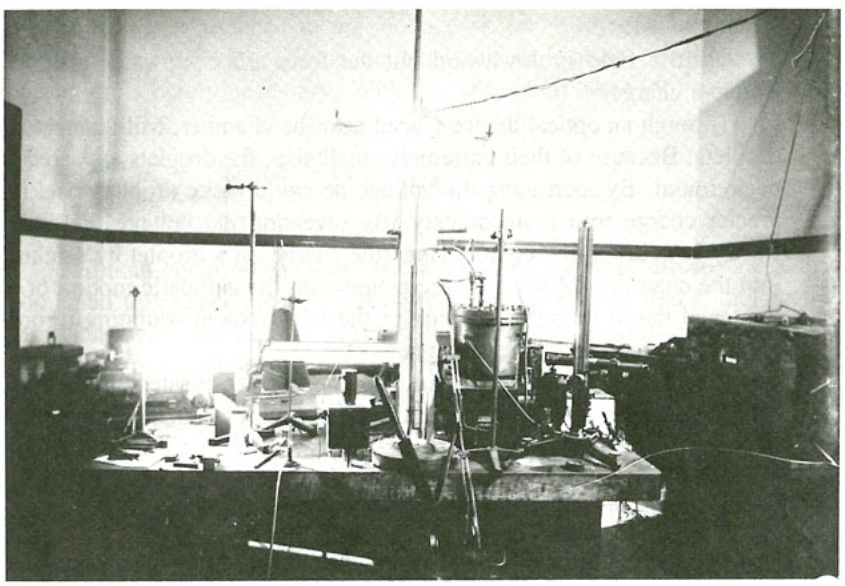

Figure 12.5 Robert A Millikan's original apparatus to measure the electron charge. (Courtesy of the Archives, California Institute of Technology.)

Millikan's original apparatus is shown in Fig. 12.5. Millikan used oil droplets for the very same reason mankind spent 300 years improving clock oils: oil droplets scarcely evaporate. Unlike water droplets, the mass of an oil droplet does not change with time. Sprayed from an atomizer, the droplets would acquire a charge due to friction as they passed through the nozzle. The charged droplets fell through a hole in one of two metal plates, which Millikan connected to a room full of electric batteries. While between the plates the droplets experience an electric force in addition to gravity, as shown in Fig. 12.6. By adjusting the voltage on the plates (and hence the electric field) certain droplets could be suspended when the upward electric force equaled the weight of the droplet:

$$qE = mg.$$

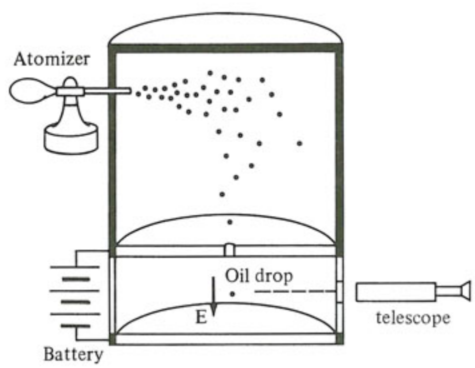

Figure 12.6 Schematic of Millikan's oil-drop apparatus.

The electric field is downward but the force qE is upward on those droplets having negative charge.

Through an optical device placed near the chamber, Millikan could watch individual droplets. Because of their extremely small size, the droplets appeared as stars on a black background. By increasing the voltage he could make droplets rise; those droplets with greater charge rose more quickly. By reversing the voltage, he could make them fall faster. In addition, he could change the charge on a droplet by sending a stream of ions into the chamber. Millikan's fascination with the acrobatic motion of droplets may have lightened the long, solitary hours in the lab he spent squinting through the device and recording hundreds of measurements.

What measurements did Millikan make that revealed the charge of the electron? Using Newton's second law, the motion of a droplet drifting upward between the plates of Fig. 12.6 can be described by

$$m\frac{dv}{dt} = qE - mg - 6\pi R\eta v. \tag{12.5}$$

The electric force pushes the negatively charged droplets upward but the viscous force is downward (opposite v) just like gravity.

By setting $dv/dt = 0$ in Eq. (12.5), we find the terminal velocity to be

$$v_1 = \frac{qE - mg}{6\pi R\eta} . \qquad (12.6)$$

The characteristic time to reach this terminal velocity turns out to be the same as when there is no electric field and is given by

$$t_0 = \frac{m}{6\pi R\eta} . \qquad (12.4)$$

A typical droplet approaches terminal velocity rapidly (in about 10^{-5} s).

Using a stopwatch to time a droplet moving between marks etched in the optical device, Millikan could measure the terminal velocity. As Eq. (12.6) indicates, the larger the charge on a droplet, the greater its terminal velocity. By observing the motion of the hundreds of droplets with different charges on them, Millikan uncovered the pattern he expected: the charges were multiples of the smallest charge he measured.

Measuring v_1 and knowing E, η, and g, you might think that Eq. (12.6) could be solved for the charge q on any droplet. But there are two other unknown quantities in that equation: the droplet's mass m and its radius R. These quantities, however, are not independent, but are related by the density of oil ρ. The oil drop has volume $\frac{4}{3}\pi R^3$, so its mass is

$$m = \tfrac{4}{3}\pi R^3 \rho.$$

To be very precise, Millikan actually used $\rho - \sigma$ in place of ρ, where σ is the density of air. The reason for this is that the air provides an additional upward buoyant force on a droplet which is equal to the weight of the air displaced by the droplet (this is known as Archimedes's law). The weight of air displaced is just the density of air times the volume of the droplet. Accounting for this force is equivalent to saying that the weight of the droplet is reduced to $mg - m_a g = (m - m_a)g$, where m_a is the mass of air displaced: $m_a = \frac{4}{3}\pi R^3 \sigma$. Since the density of air is about one-thousandth that of oil, the correction is barely necessary. With this correction taken into consideration, Eq. (12.6) becomes

$$v_1 = \frac{qE - \frac{4}{3}\pi R^3(\rho - \sigma)g}{6\pi R\eta} . \qquad (12.7)$$

Millikan could not measure the radius R of a droplet directly because the drops are too small to be seen clearly. But he had a clever way to find R indirectly. What he did was first measure the terminal velocity of a droplet drifting upward in the electric field, and then measure the terminal velocity of the same droplet falling without the electric field on. With the field on, the terminal velocity is given by Eq. (12.7). When the droplet is simply falling under the force of gravity, the terminal velocity is given by

$$v_2 = \frac{mg}{6\pi R\eta} , \qquad (12.3)$$

which when corrected for the buoyant force of air (just as we did earlier) becomes

$$v_2 = \frac{2(\rho - \sigma)R^2 g}{9\eta} . \qquad (12.8)$$

In effect, this second measurement is used to find the size of the drop, R.

From (12.7) and (12.8) we find

$$v_1 + v_2 = \frac{qE}{6\pi R\eta},$$

which, when solved for q, gives us

$$q = \frac{6\pi R\eta}{E}(v_1 + v_2).$$

But we can express R in terms of v_2 from Eq. (12.8) and we find, after some algebra, that

$$q = \frac{18\pi\eta^{3/2}}{E\sqrt{2g(\rho - \sigma)}} v_2^{1/2}(v_1 + v_2). \tag{12.9}$$

By measuring v_1 and v_2 Millikan determined the charge of a droplet.

Shown in Fig. 12.7 is a page from Millikan's research notebook, dated Friday, 15 March 1912. The column labels G and F refer to the times for a droplet to move between calibration marks in gravity alone and in the electric field, respectively. Through hundreds of such delicate measurements, Millikan, the patient experimentalist, discovered that the charge on a droplet always comes out an integral multiple (i.e., 1, 2, 3, etc.) of the smallest charge he found. Here was the first evidence that charges come in integral multiples of a fundamental charge – the charge of the electron.

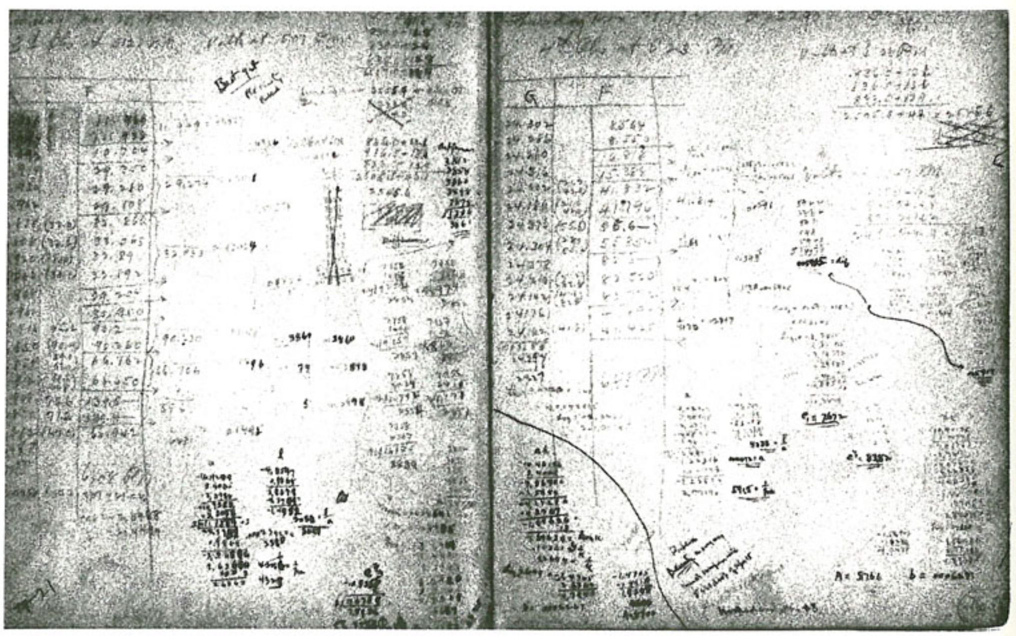

Figure 12.7 Page from Robert A. Millikan's research notebook. (Courtesy of the Archives, California Institute of Technology.)

By reevaluating the coefficient of viscosity for air and reducing errors caused by temperature variations and air currents, Millikan succeeded in determining the charge e of the electron with an error of 0.1%. The value he published in 1913 was $e = -(4.774 \pm 0.005)$ electrostatic units, equivalent to $e = -(1.603 \pm 0.002) \times 10^{-19}$ C, which served physics for a generation and is within the experimental-error bounds he gave of the most recent value. Millikan had observed the electron itself, and for his momentous efforts he received the Nobel prize in 1923.

Today physicists are searching for fractionally charged particles called quarks. Based on a symmetry classification for elementary particles, quarks are thought to be the building blocks of particles that exist inside nuclei and carry charges of $+\frac{2}{3}e$ and $-\frac{1}{3}e$. Modifications of Millikan's historic experiment are used by some of these quark hunters.

Questions

12. Fill in the steps leading to Eq. (12.9).

13. Suppose that in Millikan's oil-drop experiment it were possible to replace all the excess electrons on a given droplet with particles having twice the charge of an electron and twice the mass. Which of the following statements is true?

- **(a)** In order for the particles to be suspended between the capacitor plates, the voltage would have to be doubled.
- **(b)** With no electric field on, the drops would fall with a terminal velocity twice that of droplets having electrons.
- **(c)** The characteristic time to reach terminal velocity would be doubled.

14. Using Eq. (12.9) what percentage of error would be introduced if the correction for the buoyant force of air were not included? To find the answer, compare the charge obtained for a given v_1, v_2, etc., with and without σ considered.

15. Suppose that the electric field was such as to create a force downward rather than upward on a droplet. What then would replace Eq. (12.5)? What would be the terminal velocity in this case?

16. Show that $v(t) = t_0(qE/m - g)(1 - e^{-t/t_0})$ is a solution of Eq. (12.5), where t_0 is given in Eq. (12.4). Prove that it is consistent with the terminal velocity given in Eq. (12.6).

17. Why do you expect the characteristic time for a droplet to reach terminal velocity to be the same with or without the electric field on?

18. Estimate t_0 and v_2 in Millikan's experiment. A typical drop had a radius $R \approx 10^{-6}$ m.

12.4 A FINAL WORD

When Millikan made his measurements, alone in his laboratory, he had to have a notebook like any scientist to record what he had done. Afterward, he would gather his results together, write a scientific paper, and publish it for all the world to see. But his notebooks, the raw data of his experiments, were for his own eyes only. Figure 12.7 shows a page from Millikan's notebook. Before we criticize what we see, let's remember what Millikan was doing. He was measuring, for the first time ever, one of the fundamental constants of nature. His task was to make his measurements in the most careful, dispassionate way possible, then publish all of his results so that other scientists could judge whether he'd done it properly. The page in Fig. 12.7 is dated 15 March 1912. Here he writes down the temperature and barometric pressure, then he starts recording data, the times for a droplet: F means in the field, and G means in gravity. Then he calculates the velocities, uses logarithms to multiply them together (he didn't have a hand calculator), and finally he gets his result.

On one page he writes: "One of the best ever . . . almost exactly *right*." – What's going on here? How can it be right if he's supposed to be measuring something he doesn't *know*? On another page he writes: "Beauty. Publish!" One might expect him to publish everything! On another page, the usual stuff, then: "4% too low – something wrong." Not 4% too low but publish anyway, like a good scientist. Then something very revealing: ". . . distance wrong." He's found an excuse for not publishing it. More pages: "Beauty, one of the best," and so on for pages and pages.

Now, you shouldn't conclude that Robert Millikan was a bad scientist. He wasn't – he was a great scientist, one of the best. What we see instead is something about how real science is done in the real world. What Millikan was doing was not cheating. He was applying scientific judgment. He had a pretty clear idea of what the result ought to be – scientists almost always think they do when they set out to measure something. So, when he got a result he didn't like, he wouldn't just ignore it – *that* would be cheating. Instead, he would examine the experiments to see what went wrong. Now that seems reasonable, but it's actually a powerful bias to get the result he wants, because you can be sure that when he got a result he liked, he didn't search as hard to see what went right. But experiments must be done in that way. Without that kind of judgment, the journals would be full of mistakes, and we'd never get anywhere. So, then, what protects us from being misled by somebody whose "judgment" leads to a wrong result? Mainly, it's the fact that someone else with a different prejudice can make another measurement. Every scientist believes there *is* a real answer; it's part of nature. That's the belief that keeps scientists rigorously honest, causing them to temper and guard against their own prejudices. Dispassionate, unbiased observation is supposed to be the hallmark of the scientific method. Don't believe everything you read. Science is a difficult and subtle business, and there is no method that assures success.