PHSC 2105 **Forces and Torques** **Name:**______________________

Object: Learn about vectors: forces and torques.

Theory: The mathematics of real numbers does not describe the addition or multiplication of forces and torques. Both quantities are vectors, which means they have magnitude and direction, and we must follow rules for vector addition and multiplication.

Part I Vectors: When several forces act simultaneously on the same object their vector sum or net force can be calculated theoretically using the rules of vector addition using either of these methods:

1. By diagram (graphically): Several vectors representing simultaneous forces on an object can be added by diagramming the vectors head-to-tail using rulers and protractors to measure the magnitude and direction of each vector. The resultant vector is found by drawing an arrow from the tail of the first vector in the series to the head of the last.

2. By component (numerically): The resultant of several concurrent vectors can also be found by breaking each vector (of magnitude r and direction θ) up into its x- and y-components.

$$x = r\cos\theta \qquad y = r\sin\theta \tag{1}$$

   We measure θ from the positive x-axis going in the CCW direction for positive angles and the CW direction for negative angles. Corresponding components of the several forces are then added together separately to obtain the x- and y-components of the resultant. The magnitude and direction of this resultant vector are given by:

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{2}$$

Experimental Results: Using a force table, set up a couple of forces like those described below. Determine an additional force so as to balance the resultant sum of the two forces. This balancing force, which you will find by trial and error, is the *reverse* of the resultant force you seek.

$$\vec{F}_1 = 110\,\mathrm{g} \text{ at } 20^\circ \qquad \vec{F}_2 = 100\,\mathrm{g} \text{ at } -75^\circ$$

Theoretical Results:

1. Record the magnitude and direction of the resultant force predicted by the *diagram* method. Use graph paper to diagram the same two forces; place them head to tail and then draw the sum from the tail of the first to the head of the last.

2. Record the magnitude and direction of each resultant force predicted by the *component* method. Use Eqn. 1 to get the x- and y-components for each vector, and after summing them use Eqn. 2 to find the magnitude and direction of the sum.

Analysis: Find the percent difference between the experimental vector *magnitude* r (not θ) obtained on the force table and those obtained by each theoretical method.

Part II Torques: A *torque* is the rotational equivalent of a force. A torque results from a force $\vec{F}$ acting at a displacement $\vec{r}$ from a specific point; torques produce or tend to produce rotational or angular acceleration about the point. Torque is a vector quantity; given by

$$\vec{\tau} = \vec{r} \times \vec{F}; \tag{3}$$

its magnitude is given by $\tau = rF \sin\theta$ where θ is the angle between $\vec{r}$ and $\vec{F}$, and its direction by the right hand rule. The angle θ will be 90° for the torques in this experiment, so $\sin\theta = 1$. The official S.I. unit for torque is N·m, but it would be convenient to use units of g·cm in this experiment.

A body is said to be in *equilibrium* if there is no net force and no net torque acting on the body. Stated mathematically, a body is in equilibrium if

$$\Sigma\vec{F} = 0 \quad \text{(translational equilibrium)} \qquad \text{and} \qquad \Sigma\vec{\tau} = 0 \quad \text{(rotational equilibrium).} \tag{4}$$

Rotational equilibrium implies that the magnitude of the sum of the counter clockwise torques is equal to the magnitude of the sum of the clockwise torques: $\Sigma\tau_{\text{CCW}} = \Sigma\tau_{\text{CW}}$.

The *center of gravity* of an object is defined as the point at which a single upward force can balance the gravitational attraction on all parts of the object. Effectively, the entire mass of the body can be assumed to act at the center of gravity.

Procedure:

1. Use a laboratory balance to find the mass of a meter stick.
2. Suspend the meter stick in a knife-edge clamp and carefully slide it back and forth until it balances. Record the corresponding marking on the meter stick as the center of gravity.
3. Now suspend your meter stick at the 30.0 cm mark. Put a clamp at the 5.0 cm mark and hang enough mass on a hanger there to achieve equilibrium. Record the total amount of mass (including clamp and hanger) at the 5.0 cm mark on your accurate and clear diagram.
4. Compute the mass of the meter stick by assuming the counterclockwise torques equal the clockwise torques when the stick is in equilibrium (remember, the torque due to the weight of the meter stick is calculated as if all of the meter stick's mass were concentrated at its center of gravity).
5. Compute the percent difference between the mass of the meter stick as derived from the equilibrium equations and its mass from the laboratory balance.
6. Fold an eighth-sheet of paper in half a couple of times and place it on the meter stick at the 99 cm mark and note the results.

Discussion Questions:

1. In a couple of procedures you had to include the torque due to the weight of the meter stick itself; in others you didn't; why?

2. Could you use a modified "meter stick balance" to measure very small masses (thousandths of a gram)? How?

Conclusions: What do you conclude about the vector nature of forces and torques? What things in real life are in equilibrium?