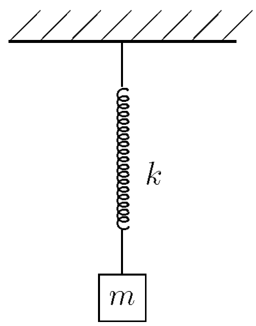
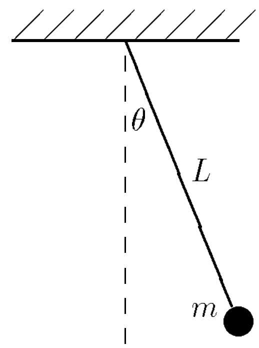
PHSC 2105 **Simple Harmonic Motion** **Name:**______________________

Object: Understand what causes harmonic motion and determine which physical variables affect the period of a harmonic oscillator and the mathematical relationship between variables.

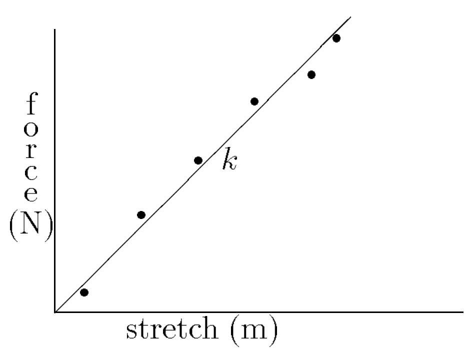
Theory: Periodic motion is motion that repeats after a time T (the period). Simple harmonic motion is a special kind of periodic motion. It is of great interest and importance in physical science because it occurs in many mechanical, electrical, and atomic systems collectively called harmonic oscillators.

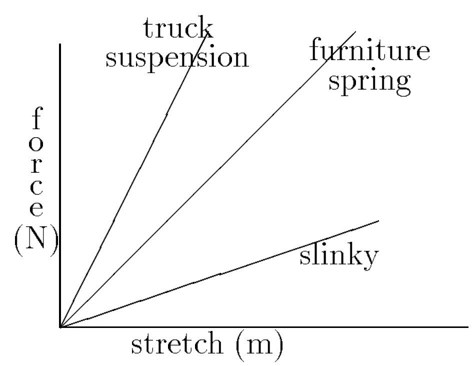
We will study two simple examples of simple harmonic motion: (1) an object with mass m oscillating at the end of a spring of stiffness k, and (2) a bob of mass m swinging back and forth at the end of a cord of length L starting at angle θ to form a simple pendulum.





The cause of simple harmonic motion is a restoring force directed back toward an equilibrium position; the strength of the force is proportional to the displacement Δs from the equilibrium. For example, when a spring is stretched or compressed it exerts a force, that is proportional to the stretch, back toward the equilibrium position. This is known as Hooke's law, and for springs it takes the form $F_{\mathrm{sp}} = -k(\Delta s)$. The negative sign merely indicates that it is a restoring force, directed back toward the equilibrium position. The constant of proportionality k is called the spring constant; it indicates how much force needs to be applied to produce a given amount of displacement Δs from the equilibrium length. One would find k for a spring by graphing the force vs. stretch for various amounts of mass; k is the slope of the straight line. A slinky would have a very small k while a furniture spring would have a higher value of k.





There is a version of Hooke's law and a restoring force for all simple harmonic oscillators.

Dimensional analysis is a technique to deduce the mathematical form of a relationship between variables by examining their fundamental physical dimensions. For example, examining Hooke's law (above) reveals that the SI units of k are N/m. We use $[X]$ to mean "the dimensions of X"; so that the dimensions of k are $[k] = [\text{force}]/[\text{length}]$ or [F]/[L]. After experimentally deciding which variables affect the period, use dimensional analysis to guess the mathematical relationship between the variables. Dimensional analysis won't give you dimensionless constants, such as τ, the circle constant, which shows up in the formulas for the period of harmonic oscillators.

<u>Procedure and Results:</u> Measure periods by letting the system oscillate for several periods and dividing the total time by the number of oscillations. Vary only one variable at a time. Make tables of your data. Label all graphs well. Think about what the shape of your graph (of T vs. some variable) would tell you. What would it mean if your graph were a horizontal line? What would it mean if your graph were a slanted straight line? What would it mean if your graph gently curved over to the right? What would it mean if T decreased as the other variable increased?

For the mass on the spring:

1. Determine by trial and error which variables in the diagram affect the period of a mass on a spring harmonic oscillator; i.e., separately vary k and m while holding the other one constant.

2. To vary k while saving time, each student will study only one spring, but each spring will have a different k. Since each spring has its own unique spring constant k, you will need to know the spring constant of your spring, as well as the other springs your classmates use. Then you can determine the dependence of the period on k by examining the data from other students for a specific constant value of m, say 550 g.

spring:	Red	Blue	Yellow	White	Green
k (N/m)	25	30	35	40	50

3. Measure the period of oscillation for your spring when you suspend the agreed m from it. Gather data on other springs from your classmates for that specific value of m.

4. Using graph paper, plot T vs. k (holding m constant). Try to identify the shape of the graph to help you in your dimensional analysis.

5. Now hang different masses on your spring (that is, vary m, while keeping k constant) and measure the period T for small oscillations. Do this by beginning with $\approx$ 350 g (remember, the hanger has a mass of 50 g) and increasing by 50 g each trial for about five or six trials.

   You will get better results if you record the hanging mass plus 1/3 the mass of the spring on each line of the table. The reason is that ideal springs are massless and yours aren't.

6. Using graph paper, plot T vs. m. Try to identify the shape of the graph to help you in your dimensional analysis.

7. From the shape of the graphs and by analyzing the dimensions of m and k try to figure out the functional dependence of period on mass and the spring constant. That is, write a formula for the period of this harmonic oscillator where T is a function of m and k. Both sides of the equation must have dimensions of time: [T].

For a simple pendulum:

1. Using a bob and string, form a simple pendulum. By trial and error decide which variables in the diagram affect the period.

2. Vary L; keep m and θ constant. Measure the period (for small oscillations, $\theta < 20°$) for different lengths L (measured to the middle of the bob) of the pendulum. Start with a length of 20 cm and increase by 20 cm each trial until maximum length is reached.

3. Make a graph of period vs. length. Try to identify the shape of the graph to help you in your dimensional analysis.

4. Vary m; keep L and θ constant. Measure the period (for small oscilations, $\theta < 20°$) for different masses m of the pendulum. Remember that L is measured to the middle of the bob.

5. Make a graph of period vs. mass. Try to identify the shape of the graph to help you in your dimensional analysis.

6. Vary θ; keep L and m constant. Measure the period for different starting angles θ of the pendulum. Use angles of 5°, 10°, 15°, 20°, 30°, 40°, 50°, 60°, 70°, and 80°.

7. Make a graph of period vs. swing angle. Try to identify the shape of the graph to help you in your dimensional analysis.

8. From the shape of the graphs and by analyzing the dimensions of m, L, and θ try to figure out the functional dependence of period on mass, length, and swing angle. That is, write a formula for the period of this harmonic oscillator where T is possibly a function of m, L, and θ. Both sides of the equation must have dimensions of time: [T]. (Hint: Only a small angle pendulum is considered to be a simple harmonic oscillator.)

The shape of the graphs and the dimensional analysis will give you the functional dependence of the period on the other variables (this is also called a scaling law). But they won't give you the dimensionless constant, $\tau = 2\pi$, which appears in both of them. (Not just the shape, but accurate analysis of the height of the graphs would show the τ.)

<u>Conclusions:</u> Explain how to find the dependence of one physical quantity on others. How does graphing help this process?